11 Similarity. Thales and Pythagoras theorems

LAND HO!

Land ho! This was the sailor’s cry that satisfied Columbus crew.

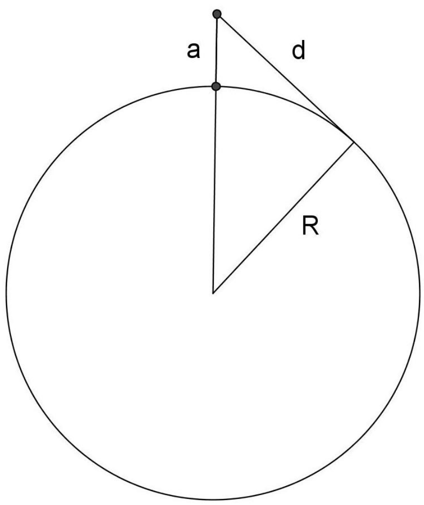
You may have observed on the beach that the distance you can see depends on the height you have over the horizon.

That is the reason why the sea seems wider if you are on a cliff than if you are on the seaside.

But, what is the relationship between the height of a point and the visible distance from it?

We begin with a picture to understand the problem.

The circle is the Earth, R is the radius (about 6000 km), *'a'* is the height we have over sea level and *'d'* is our visible horizon.



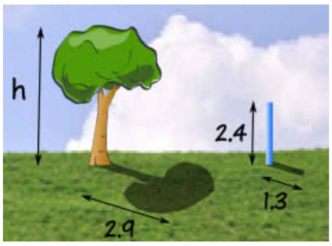
Could you calculate your visible horizon from a height of 30m? As you suppose you have to use Pythagoras! Ready?

1. SIMILAR FIGURES

Two figures are similar if they have the same shape but different size.

For instance, a map and the reality, a plan and the reality, a model and the reality …

That means they have the same angles and proportional sides.

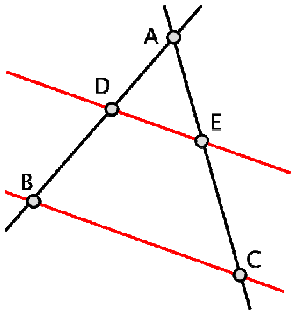


The ratio between two like sides is constant, this is named the *ratio of similarity or* scale. A map 1:200 scale means 1 unit from the map is 200 units from the reality.

The sun produces many similar figures if we compare the height and the shadow of an object.

Calculate the height of the tree from the data in the picture.

2. THALES THEOREM



A theorem is a discovery we get by reasoning.

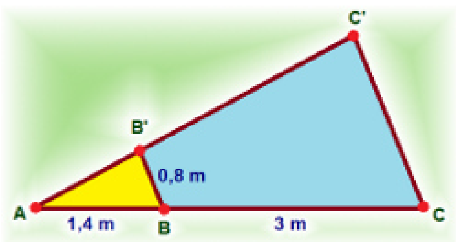
There are two very important theorems in Geometry: Thales theorem and Pythagorean theorem.

Thales of Miletus was a wise man from Ancient Greece (VI century BC)

The property described by Thales about Geometry states the following:

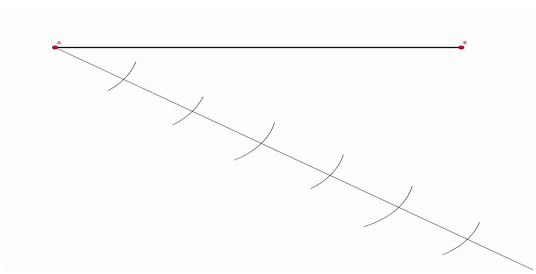
Parallels through several straight lines form proportional segments.

That means the triangles we get are similar: they have equal angles and proportional sides.



If two triangles have a common angle and parallel opposite sides to the angle then they are similar. This property is very useful to solve many problems.

Work out the length CC' from the data in the picture



Thales theorem provides a method to get similar figures by projection from a point and using parallel lines as well as to divide a segment into proportional parts to others.

Divide the segment into 6 equal parts using the Thales theorem.

3. RELATIONS BETWEEN SIMILAR FIGURES

LENGHS, AREAS AND VOLUMES

If the length ratio is 'r' the area ratio is 'r^2' and the volume ratio is 'r^3'

A 1 m tall child could weigh 12 kg (he or she could be a 4 years of age) but a 2 m tall man weighs much more than 24 kg because the increasing length is three-dimensional, that is, the increase of a volume. Because the length ratio is 2 the volume ratio will be $2^3 = 8$. That means the reasonable weight will be $8 \cdot 12 = 96$ kg.

SCALES

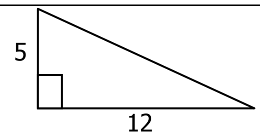
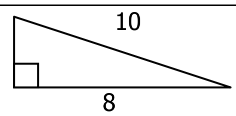
The scale is the ratio between two similar figures. We mostly use this concept speaking about maps, plans and models… The usual way to write this figure is 1 : n. The meaning is '1 unit of the chart is equivalent to 'n' units of the reality.

A plan is a representation whose scale is greater than 1:10000.

A map is a representation whose scale is lesser than 1:10000.

A model is a three-dimensional representation.

4. PYTHAGORAS THEOREM

The sides of a right triangle are called: hypotenuse (h) and catheti or legs (a and b). The theorem establishes that: $h^2 = a^2 + b^2$ Three numbers that verify the above equality are called Pythagorean triple.	
Calculate the unknown side	
Calculate the unknown side	

If we draw the height from hypotenuse of a triangle we can find a lot of similarity relations among the three triangles we get: the total triangle and the two newly created triangles.

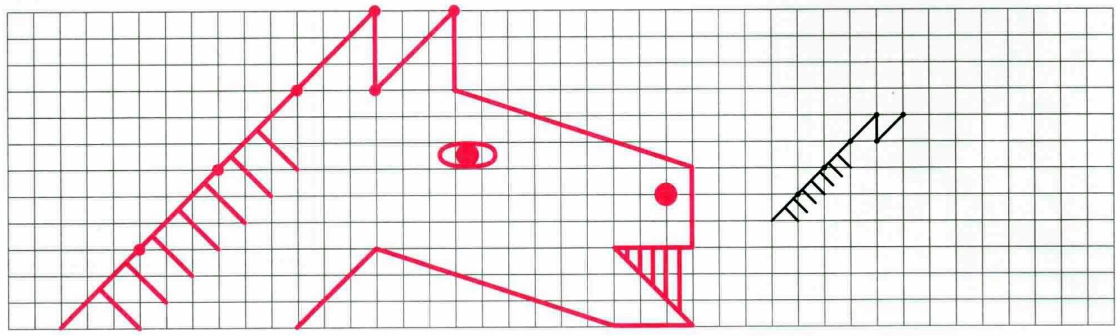
If we compare the two new triangles we will deduce the height theorem.

If we compare each triangle with the total we will deduce the catheti theorem.

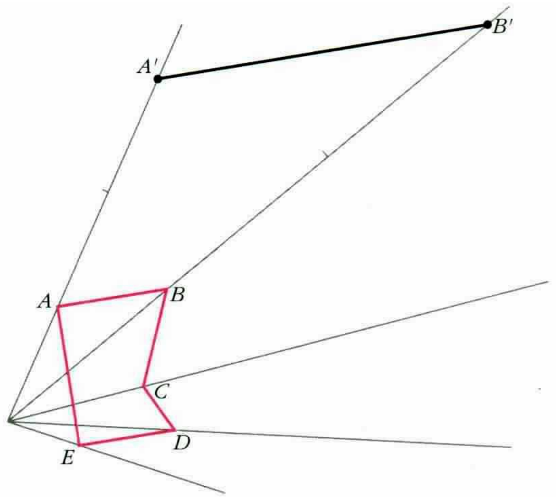
PROBLEMS AND EXERCISES

1. SIMILAR FIGURES

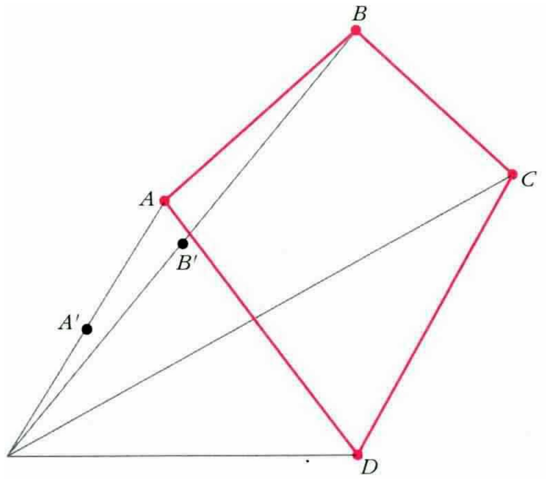
1. Reduce the shape to one-third of its present size



2. Enlarge the shape to 3 times its present size



3. Reduce the shape to one-half of its present size



2. THALES THEOREM

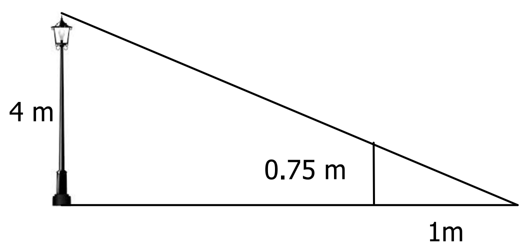
4. A tourist 1.80 m tall admires this amazing sculture.
What is the length of the sculpture approximately?
What is the scale of the picture?

5. We want to divide a segment 10 cm long into three parts directly proportional to 3 cm, 8cm and 9 cm. How long will each new segment be?

6. A monitor is 1280 x 1024 pixels. What is its format 4:3 or 16:9?

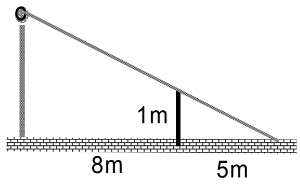
7. A stick 80 cm long produces a shadow of 30 cm. What is the height of a building whose shadow is 6m at the same time?

8. A stick 0.75 m long produces a shadow 1m long.
Knowing the streetlamp is 4m long, what is the distance between the stick and the streetlamp?



9. A stick 1 m long is placed 8m from a streetlight and produces a shadow 5m long.

Taking the data from the picture, what is the hight of the streetlamp?



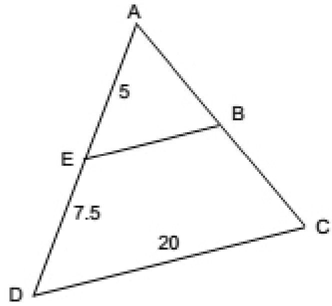
10. The drawing shows two similar triangles with a common vertex A.

a. Determine the length of EB.

b. What is the similarity ratio between the two triangles?.

c. Knowing that AB=AE, that is, the triangle AEB is isosceles, what is the area of this triangle?

d. And finally, what is the area for the largest triangle. That is ADC triangle.



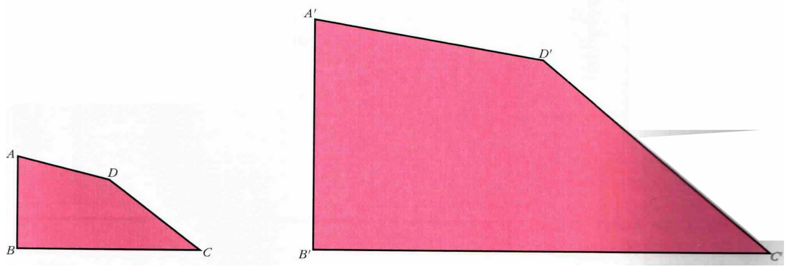
11. We want to divide a segment 30 cm long into three parts directly proportional to 3 cm, 8cm and 9 cm. How long will each new segment be?

3. RELATIONS BETWEEN SIMILAR FIGURES

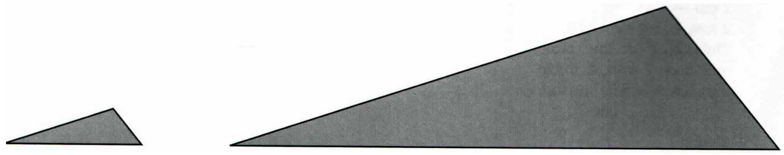
12. These two shapes are similar. Measure and compare their sides.

What is the similarity ratio?

What is the surface ratio?



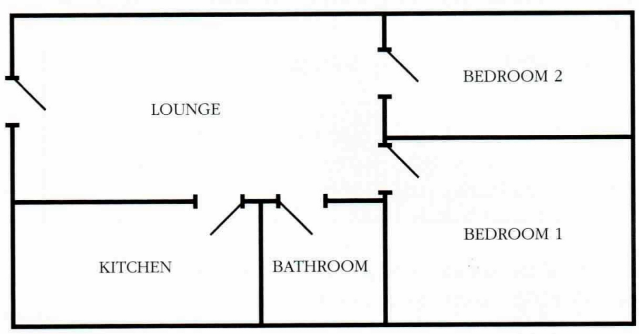
13. Take measurements and write down the similarity ratio of the second triangle with respect to the first. What is the surface ratio?



14. This is the floor plan of a flat. It is drawn to a scale of 1:400. This means that the actual lengths are really 400 times bigger than they appear on the plan. In other words, 1 cm on the plan corresponds to 4 metre in the real life.

a. Calculate the dimensions of the entire flat as well as of the rooms.

b. Calculate the surface of the plan and the real surface.



15. A cuboid shaped package is 8 cm long, 25 cm high and 5 cm wide.

a) Calculate the total surface area.

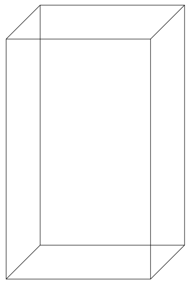
b) Calculate the volume.

A similar package is 10 cm long.

c) What is the similarity ratio?

d) Calculate the dimensions for this new package, that is, the height and the width.

e) Calculate the surface ratio and the volume ratio.



16. A cuboid shaped package is 4 cm long, 25 cm high and 5 cm wide.

a) Calculate the total surface area.

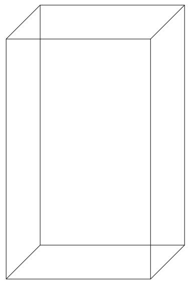
b) Calculate the volume.

A similar package is 8 cm long.

c) What is the similarity ratio?

d) Calculate the dimensions for this new package, that is, the height and the width.

e) Calculate the surface ratio and the volume ratio.



17. You have a small marble statue of Wolfgang Mozart that is 4 dm tall. The original statue in Vienna is 2 m tall.

a. What is the similarity ratio?

b. The width of your statue is 40 cm, what is the real width?

c. What is the surface ratio between the two statues?

d. Your statue has a volume of 2.4 dm^3. Estimate the volume of the original statue.

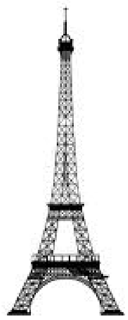
Wolfgang Mozart

18. You have a small model of the Eiffel Tower whose scale is 1:2000. Your model is 16.5 cm tall

a. What is the real Eiffel Tower height? Give your result in meters

b. What is the area ratio between the two statues?

c. Your statue has a weight of 2.5 kg. If the material is the same in both towers what would be the real weight? Notice the weight is proportional to the volume. Express the weight in tonnes.



19. A map has a scale of 1:80 000 000. What real distances are represented by 1 cm on the map? And by 4.8 cm? What is the real surface of a 1 cm^2 square?

20. A map has a scale of 1 : 40 000. What real distances are represented by 1 cm on the map? And by 4.8 cm?

What is the real surface area of a 1 cm^2 square?

4. *PYTHAGORAS'S THEOREM*

21. Calculate the hypotenuse of a right-angled triangle whose other sides measure 30 cm and 16 cm.

22. The legs of a right-angled triangle measure 15 cm and 20 cm. Find the length of the hypotenuse.

23. The hypotenuse of a right-angled triangle measures 34 dm and one of its legs measures 30 dm. Work out the other leg.

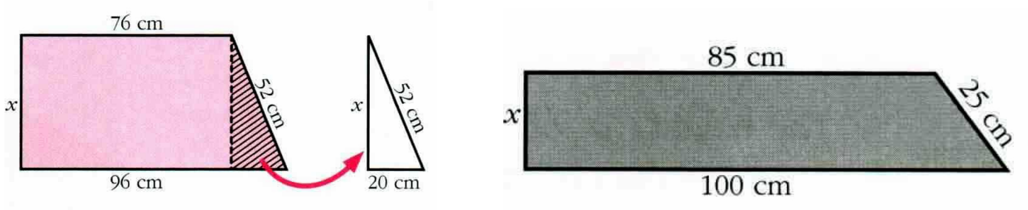
24. A television is 4:3 format and the diagonal is 20 inches. What are the length and width?

25. Find the height of an isosceles triangle whose base measures 20 cm and whose equal sides measure 26 cm each.

26. The perpendicular height to the unequal side of an isosceles triangle is 30 cm and this side is 32 cm. Find the length of the two equal sides.

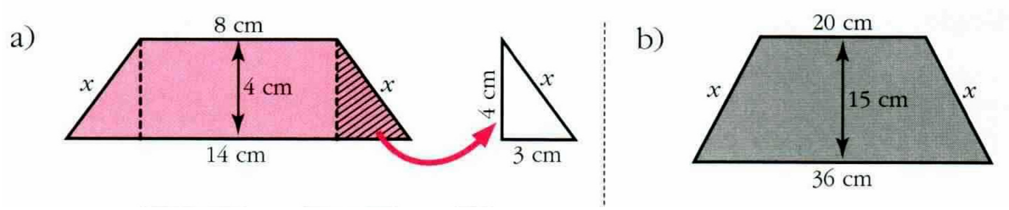
27. The two equal sides of an isosceles triangle measure 50 cm and the perpendicular height to its unequal side is 38 cm. Find the length of this side.

28. Find the length of the unknown side of this right-angled trapezium. Do the same for the other.



29. The bases of a right-angled trapezium measure 20 m and 38 m. Its height is 13 m. Calculate its perimeter.

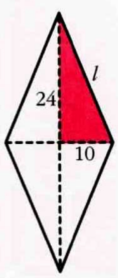
30. Find the lengths of the unknown sides of this isosceles trapezium



31. The bases of an isosceles trapezium measure 23 cm and 58 cm. The two equal sides measure 21 cm. Calculate its height.

32. Calculate the length of the sides of a rhombus with known diagonals of *d* and *d'*.

a) $d = 48$ cm, $d' = 20$ cm



$$l = \sqrt{24^2 + 10^2} = \sqrt{576 + 100} =$$

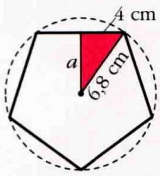
$$= \sqrt{676} = 26$$

The sides measure 26 cm.

b) $d = 90$ cm, $d' = 4$ dm

33. Calculate the apothem of a regular polygon with a known side of *l* and a radius of r.

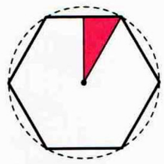
a) Pentagon: $l = 8$ cm, $r = 6.8$ cm



$$a = \sqrt{6.8^2 - 4^2} = \sqrt{46.24 - 16} =$$
$$= \sqrt{30.24} = 5.5$$

The apothem measures 5.5 cm.

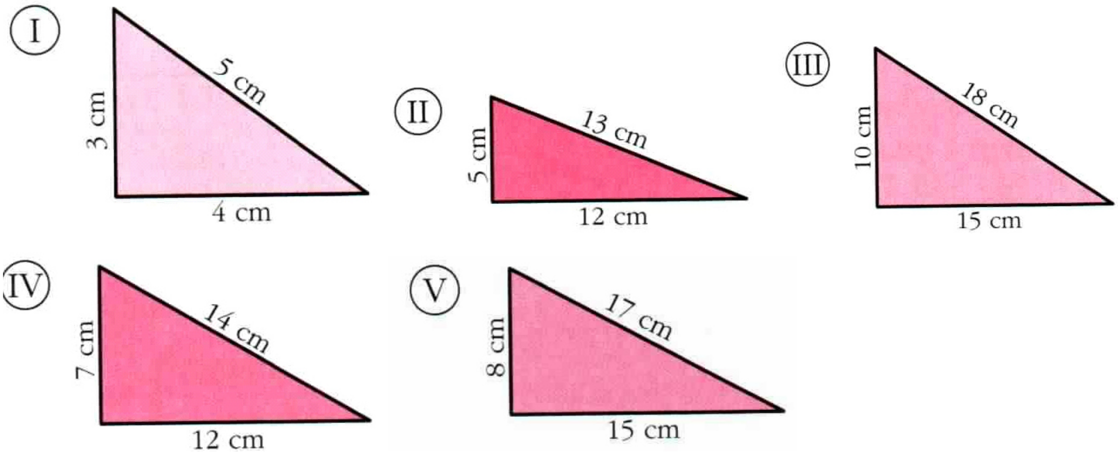
b) Hexagon: $l = 15$ cm, $r = 15$ cm



34. We know that one diagonal of a rhombus is 80 cm and the length of its sides is 62 cm. Calculate the length of the other diagonal.

35. The side of a regular octagon measures 10 cm and its apothem is 12 cm. What is the radius?

36. Determine whether the following triangles are right-angled triangles by Pythagoras. The picture isn't accurate to reality. Could you tell what kind of triangle is each according to its angles?



37. Find the length of the unknown side for each of the following right-angled triangles. If the result is not integer, express it as a decimal number.